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Can Spacetime be a Condensate?

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We explore further the proposal [Hu, B. L. (1996). General relativity as geometro-hydrodynamics. (Invited talk at the Second Sakharov Conference, Moscow, May 1996); gr-qc/9607070.] that general relativity is the hydrodynamic limit of some fundamental theories of the microscopic structure of spacetime and matter, i.e., spacetime described by a differentiable manifold is an emergent entity and the metric or connection forms are collective variables valid only at the low-energy, long-wavelength limit of such micro-theories. In this view it is more relevant to find ways to deduce the microscopic ingredients of spacetime and matter from their macroscopic attributes than to find ways to quantize general relativity because it would only give us the equivalent of phonon physics, not the equivalents of atoms or quantum electrodynamics. It may turn out that spacetime is merely a representation of certain collective state of matter in some limiting regime of interactions, which is the view expressed by Sakharov [Sakharov, A. D. (1968). *Soviet Physics-Doklady* **12**, 1040–1041; Sakharov, A. D. (1967). Vacuum quantum fluctuations in curved space and the theory of gravitation. *Doklady Akad. Nauk S.S.R.* **177**, 70; Adler, S. L. (1982). *Reviews of Modern Physics* **54**, 729]. In this talk, working within the conceptual framework of geometro-hydrodynamics, we suggest a new way to look at the nature of spacetime inspired by Bose–Einstein condensate (BEC) physics. We ask the question whether spacetime could be a condensate, even without the knowledge of what the 'atom of spacetime' is. We begin with a summary of the main themes for this new interpretation of cosmology and spacetime physics, and the 'bottom-up' approach to quantum gravity. We then describe the 'Bosenova' experiment of controlled collapse of a BEC and our cosmology-inspired interpretation of its results. We discuss the meaning of a condensate in different context. We explore how far this idea can sustain, its advantages and pitfalls, and its implications on the basic tenets of physics and existing programs of quantum gravity.

KEY WORDS: spacetime; condensate; BEC; general relativity; stochastic; quantum gravity; cosmology; string theory; quantum-classical; Kinetic theory; hydrodynamics.

1. INTRODUCTION

1.1. Classical, Semiclassical, Stochastic, and Quantum Gravity

The theory of general relativity provides an excellent description of the features of large-scale spacetime and its dynamics. "Classical gravity" assumes

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classical matter as source in the Einstein equation. When quantum fields are included in the matter source, a "quantum field theory in curved spacetimes" (QFTCST) is needed (Birrell and Davies, 1982; DeWitt, 1975; Fulling, 1989; Grib *et al.*, 1994; Mirzabekian and Vilkovisky, 1998; Wald, 1994). At the semiclassical level the source in the (semiclassical) Einstein equation is given by the expectation value of the energy momentum tensor operator of quantum matter fields with respect to some quantum state. 'Semiclassical gravity' (Anderson, 1983, 1984; Calzetta and Hu, 1987; Campos and Verdaguer, 1994; Fischetti *et al.*, 1979; Hartle and Hu, 1979; Hu and Parker, 1978; Zel'dovich and Starobinsky, 1971a,b) refers to the theory where classical spacetime is driven by quantum fields as sources, thus it includes the backreaction of quantum fields on spacetime and self-consistent evolution of quantum field and spacetime together. Without the requirement of self-consistent backreaction, QFTCST can be viewed as a test field approximation of semiclassical gravity. 'Stochastic gravity' (Hu, 1989, 1999; Hu and Verdaguer, 2003, 2004; Martin and Verdaguer, 1999a,b, 2000) includes the fluctuations of quantum field as source described by the Einstein–Langevin equation (Calzetta and Hu, 1994; Campos and Verdaguer, 1996; Hu and Matacz, 1995; Hu and Sinha, 1995; Lombardo and Mazzitelli, 1997). Our program on 'quantum gravity' uses stochastic gravity as a launching platform and kinetic theory (Hu, 2002) as a program guide.

To anchor our discussions, we summarize here some 'lead ideas' on the three levels of gravitation theory, shy of quantum gravity. We present the main thesis, spell out the major tasks for each level and new questions which we need to address.

1.1.1 Cosmology as 'Condensed Matter Physics

Of importance is not just 'particles and fields,' which one obviously needs for the content of matter which drives the dynamics of spacetime, but also how they organize and transform on larger scales. We emphasize the importance of ideas from 'condensed matter physics,' (in conjunction with quantum field theory, for treating early universe quantum processes) in understanding how spacetime and matter in different forms and states interplay and evolve. The suggestion of viewing cosmology in the light of condensed matter physics, in terms of taking the correct viewpoints to ask the right questions, and approaches to understand the processes, has been made earlier (e.g., Hu (1988)). Phase transition processes underlie the foundation of the inflationary cosmology program. Proposals to study cosmological defect formation in helium experiments and to view cosmology as a critical phenomenon were proposed (Smolin, 1995; Zurek, 1996). A recent monograph is devoted to the unity of forces at work in He^3 droplets (Volovik, 2003) (see also Jacobson and Koike (2002); Jacobson and Volovik (1998a,b,c)).

1.1.2 General Relativity as Hydrodynamics

In our view (Hu, 1996), general relativity is the hydrodynamic (the low-energy, long-wavelength) regime of a more fundamental microscopic theory of spacetime, and the metric and the connection forms are the collective variables derived from them. At shorter wavelengths or higher energies, these collective variables will lose their meaning, much as phonon modes cease to exist at the atomic scale. This view marks a big divide on the meaning and practice of quantum gravity. In the traditional view, quantum gravity means quantizing general relativity, and in practice, most programs under this banner focus on quantizing the metric or the connection functions. Even though the stated goals of finding a microstructure of spacetime is the same, the real meaning and actual practice between these two views are fundamentally different. If we view GR as hydrodynamics and the metric or connection forms as hydrodynamic variables, quantizing them will only give us a theory for the quantized modes of collective excitations, such as phonons in a crystal, but not a theory of atoms or QED. (A similar viewpoint is expressed by Jacobson (1995) from a different angle. See also Padmanabhan (2004).)

1.1.3 Stochastic Semiclassical Gravity: Fluctuations and Correlations

Stochastic semiclassical gravity is a consistent and natural generalization of semiclassical gravity to include the effects of quantum fluctuations. The centerpiece of this theory is the stress-energy bi-tensor and its expectation value known as the noise kernel. The key point here is the important role played by noise, fluctuations, dissipation, correlations, and quantum coherence, the central issues focused in and addressed by mesoscopic physics. This new framework allows one to explore the quantum statistical properties of spacetime: How fluctuations in the quantum fields induce metric fluctuations and seed the structures of the universe, black hole quantum horizon fluctuations, the backreaction of Hawking radiance in black hole dynamics, and implications on trans-Planckian physics. On the theoretical issues, stochastic gravity is the necessary foundation to investigate the validity of semiclassical gravity and the viability of inflationary cosmology based on the appearance and sustenance of a vacuum energy-dominated phase. It is also a useful platform supported by well-established low energy (sub-Planckian) physics to explore the connection with high energy (Planckian) physics in the realm of quantum gravity.

1.1.4 'Bottom-up' Approach to Quantum Gravity: Mesoscopic Physics

As remarked above, we find it more useful to find the microscopic variables than to quantize the macroscopic variables. If we view classical gravity as an effective theory, i.e., the metric or connection functions as collective variables of some fundamental particles which make up spacetime in the large, and general relativity as the hydrodynamic limit, we can also ask if there is a regime like kinetic theory of molecular dynamics or mesoscopic physics of quantum many-body systems intermediate between quantum micro-dynamics and classical macro-dynamics. This transition involves both the micro to macro transition and the quantum to classical transition, two central issues in mesoscopic physics. We will describe the mesoscopic physics issues here and the kinetic theory approach to quantum gravity in the next section.

In Hu (1997), we pointed out that many issues special to this intermediate stage, such as the transition from quantum to classical spacetime via the decoherence of the 'density matrix of the universe,' phase transition or cross-over behavior at the Planck scale, tunneling and particle creation, or growth of density contrast from vacuum fluctuations, share some basic concerns of mesoscopic physics in atomic or nuclear condensed matter or quantum many-body systems. Underlying these issues are three main factors: Quantum coherence, fluctuations, and correlations. We discuss how a deeper understanding of these aspects of fields and spacetimes related to the quantum/classical and the micro/macro interfaces, the discrete/continuum or the stochastic/deterministic transitions can help to address some basic problems in gravity, cosmology, and black hole physics such as Planck scale metric fluctuations, cosmological phase transition and structure formation, and the black hole entropy, end-state and information paradox.

Mesoscopic physics deals with problems where the characteristic interaction scales or sample sizes are intermediate between the microscopic and the macroscopic. The experts refer to a specific set of problems in condensed matter and atomic/optical physics. For the present discussion, I will adopt a more general definition, with 'meso' referring to the interface between macro and micro on the one hand and the interface between classical and quantum on the other. These two aspects will often bring in the continuum/discrete and the deterministic/stochastic factors. These issues concerning the micro/macro interface and the quantum to classical transition arise in quantum cosmology and semiclassical gravity in a way categorically similar to the new problems arising from condensed matter and atomic/optical physics (and, at a higher energy level, particle/nuclear physics, at the quark–gluon and nucleon interface). Similarly, many issues in gravitation and cosmology are related to the coherence and correlation properties of quantum systems, and involve stochastic notions, such as noise, fluctuations, dissipation, and diffusion in the treatment of transport, scattering, and propagation processes.

The advantage of making such a comparison between these two apparently disjoint disciplines is twofold: The theory of mesoscopic processes that can be tested in laboratories with the new nanotechnology can enrich our understanding of the basic issues common to these disciplines while being extended to the realm of general relativity and quantum gravity. The formal techniques developed and applied to problems in quantum field theory, geometry, and topology can be adopted to treat condensed matter and atomic/optical systems with more rigor, accuracy, and completeness. Many conceptual and technical challenges are posed by mesoscopic processes in both areas.

1.2. Geometro-Hydrodynamics: Spacetime as Condensate

We now present a new idea inspired by the development of Bose–Einstein condensate (BEC) physics in recent years. While the conception of mesoscopic physics and the kinetic theory approach to quantum gravity bear on the last two themes in the prior subsection, here we return to the first two themes, dealing with the hydrodynamic properties of spacetime and their manifestation in cosmology through quantum processes involving vacuum fluctuations. The idea is that maybe spacetime, describable by a differentiable manifold structure, valid only at the low-energy long-wavelength limit of some fundamental theory, is a condensate. We will devote a section examining what a condensate means, but for now we can use the BEC analog and think of it as a collective quantum state of many atoms with macroscopic quantum coherence. When this thought came to my mind some 5–6 years ago amidst bursting activities of BEC experiments and theories, I discarded it immediately for the obvious absurdities indicated below. After living with this idea for sometime I found that they do not seem as repugnant as before so I dare to share them here in the hope the audience/reader can throw some much-needed light to it.

1.2.1 Unconventional View 1: All Sub-Planckian Physics are Low-Temperature Physics

Atom condensates exist at very low temperatures. It takes novel ways of cooling the atoms, many decades after the theoretical predictions, to see a BEC in the laboratories. It may not be too outlandish to draw the parallel with spacetime as we see it today, because the present universe is rather cold (3K). But we believe that the physical laws governing today's universe are valid all the way back to the grand unification theory (GUT) and the Planck epochs, when the temperatures were not so low any more. Any normal person would consider the Planck temperature $T_{\mathrm{Pl}} = 10^{32}$K a bit high. Since the spacetime structure is supposed to hold (Einstein's theory) for all sub-Planckian eras, if we consider spacetime as a condensate today, should it not remain a condensate at this ridiculously high temperature? That was PUZZLE number 1.

YES is my answer to this question. What human observers consider as high temperature (such as that when species *Homo sapiens* will instantly evaporate) has no effect on the temperature scales defined by physical processes which in turn are governed by physical laws. Instead of conceding to a breakdown of the spacetime condensate at these temperatures, for the sake of arguments here, one should push this concept to its limit and come to the conclusion that all known physics today, as long as a smooth manifold structure remains valid for spacetime, the arena where all physical processes take place, are low-temperature physics. Spacetime condensate exists even at Planckian temperature T_{Pl}, but will cease to exist above the Planck temperature, according to our current understanding of the physical laws. In this sense spacetime physics as we know it is low-temperature hydrodynamics, and, in particular, today we are dealing with ultra-low temperature physics, similar to superfluids and BECs.[3]

The metric or connection functions are hydrodynamic variables, and most macroscopic gravitational phenomena can be explained as collective modes and their excitations (of the underlying deeper micro-theory): from gravitational waves in the weak regime as perturbations, to black holes in the strong regime, as solitons (nonperturbative solutions). There may even be analogs of turbulence effects in geometro-hydrodynamics, made apparent when our observation or numerical techniques are improved.

1.2.2 Unconventional View 2: Spacetime is, After all, a Quantum Entity

An even more severe difficulty in viewing spacetime as a condensate is to recognize and identify the quantum features in spacetime as it exists today, not at the Planck time. The conventional view holds that spacetime is classical below the Planck scale, but quantum above it. That was the rationale for seeking a quantum version of general relativity, beginning with quantizing the metric function and the connection forms. Our view is that the universe is fundamentally a quantum phenomena,[4] but at the mean field level the many-body wave functions (of the micro-constituents, or the 'atoms' of spacetime) which we use to describe its large-scale behavior (order parameter field) obey a classical-like equation, similar to the Gross–Pitaevsky equation in BEC, which has proven to be surprisingly successful in capturing the large-scale collective dynamics of BEC (Pethick and Smith, 2002), until quantum fluctuations and strong correlation effects enter into the picture (Rey *et al.*, 2004).

Could it be that the Einstein equations depict the collective behavior of the spacetime quantum fluid on the same footing as a Gross–Pitaevsky equation for BEC? The deeper layer of structure is ostensibly quantum, it is only at the mean

[3] Other discussions of condensates in gravity include the proposal of Mazur and Mottola on the existence of gravitational vacuum condensate stars Mazur and Mottola (2004) related to the earlier work of Chapline *et al.* (2001) on quantum phase transitions near a black hole horizon. We are considering the properties of post-Planckian spacetime in general terms while their considerations predict specific consequences for unknown and known astrophysical objects. While their general views in a broader perspective may be considered to be similar to what is proposed here, we cannot concur with their specific claims.

[4] This is still a nascent and very tentative view, I will explore this idea further in the context of macroscopic quantum phenomena in Hu (2006).

field level that the many-body wave function is amenable to a classical description. We have seen many examples in quantum mechanics where this holds. In truth, for any quantum system which has bilinear coupling with its environment or is itself Gaussian exact (or if one is satisfied with a Gaussian approximation description), the equations of motion for the expectation values of the quantum observables have the same form as its classical counterpart. The Ehrenfest theorem interwoven between the quantum and the classical is one common example.

The obvious challenge is, if the universe is intrisically quantum and coherent, where can one expect to see the quantum coherence phenomena of spacetime? Here again we look to analogs in BEC dynamics for inspiration, and there are a few useful ones, such as particle production in the collapse of a BEC, which we will describe in a later section. One obvious phenomenon staring at our face is the vacuum energy of the spacetime condensate, because if spacetime is a quantum entity, vacuum energy density exists unabated for our present day late universe, whereas its origin is somewhat mysterious for a classical spacetime in the conventional view. We would like to explore the implications of this view on the cosmological constant and coincidence problems later.[5]

In the next section, we summarize the 'kinetic theory approach' to quantum gravity, as it is one way to connect the (macro) hydrodynamics to the (micro) molecular dynamics.[6] In Section 4, we address the main issues associated with the spacetime condensate viewpoint, taking on its meaning and discussing its implications on the basic tenets of physics and existing programs of quantum gravity.

2. FROM STOCHASTIC TO QUANTUM GRAVITY VIA METRIC CORRELATION HIERARCHY

In this section, we summarize the main points in the kinetic theory approach to quantum gravity (Hu, 2002). Again, by quantum gravity we mean a theory of the microscopic structure of spacetime, not necessarily a theory obtained by quantizing general relativity. The key ideas utilized to construct this proposal are the correlation hierarchy (Balescu, 1975; Calzetta and Hu, 1988), decoherence of correlation history (Calzetta and Hu, 1993), correlation noise (Calzetta and Hu, 1995), and stochastic Boltzmann equation (Calzetta and Hu, 2000). In

[5] Volovik has proposed some solutions to these problems. While we agree with his general attitude we reserve our judgment on the particulars in the theories and models he proposed. The issues are subtle and complex. See also Padmanabhan's views in Padmanabhan (2002)

[6] For the last two decades working on these ideas I have been inspired by work of Boon and Yip (1991) on the relation between hydrodynamics and molecular dynamics, and Forster (1975); Kac and Logan (1979, 1976); Lax (1968) on hydrodynamic fluctuations. To see how many-atom correlations interplay with hydrodynamical features of BEC via a kinetic theory description, see, e.g., Rey *et al.* (2004, 2005)

statistical physics, it is well known that intermediate regimes exist between the long-wavelength hydrodynamics limit and the microdynamics.[7] The central task for us is the retrieval or reconstruction of quantum coherence in the gravity sector. We do this through fluctuations and correlations, starting from the matter sector described by quantum fields, and connecting to the gravity sector by the Einstein equations, at the hydrodynamic level, and its higher order hierarchical generalizations, at the kinetic theory level. The pathway from stochastic to quantum gravity [8] in the kinetic theory approach is via the correlation hierarchy of noise and induced metric fluctuations. Readers who are familiar with this can skip to the next section.

We see that stochastic semiclassical gravity provides a relation between noise in quantum fields and metric fluctuations. While the semiclassical regime describes the effect of a quantum matter field only through its mean value (e.g., vacuum expectation value), the stochastic regime includes the effect of fluctuations and correlations. We believe that precious new information resides in the two-point functions and higher order correlation functions of the stress-energy tensor which may reflect the finer structure of spacetime at a scale when information provided by its mean value as source (semiclassical gravity) is no longer adequate.

Our strategy is to look closely into the quantum and statistical mechanical features of the matter field in deepening levels and see what this implies on the spacetime structure at the corresponding levels. (This is different from the induced gravity program (Adler, 1982; Sakharov, 1987, 1968) although the spirit is similar). Thus, we work with both the microstructure of matter described by quantum field theory of matter and the macrostructure of spacetime described by hydrodynamics. We rely on higher order correlations in moving beyond the semiclassical gravity stage. The procedures in this approach involve the deduction of the correlations of metric fluctuations from correlation noise in the matter field, identifying distinct collective variables depicting recognizable metastable structures in the kinetic and hydrodynamic regimes of quantum matter fields and finding out the corresponding structure and behavior in their spacetime counterparts.

This will give us a hierarchy of generalized stochastic equations, the Boltzmann–Einstein hierarchy of quantum gravity, for each level of spacetime structure, from the macroscopic (general relativity) through the mesoscopic (stochastic gravity) to the microscopic (quantum gravity). The linkage at the lowest level is provided by the Einstein equation. Stochastic gravity entails all the higher rungs between semiclassical and quantum gravity, much like the BBGKY (Balescu, 1975) or the Dyson–Schwinger hierarchy (Calzetta and Hu, 2000) representing kinetic theory of matter fields.

[7] They are usually lumped together and called the kinetic regime, but I think there must be distinct kinetic collective variables depicting recognizable metastable intermediate structures in this vast interim regime (see Spohn (1991))

[8] For discussions on this more general issue, see, e.g., Accardi *et al*. (2002); Anastopoulos (2001); Calzetta *et al*. (2003).

2.1. Noise and Fluctuations as Measures of Correlations and Coherence

In Hu (1999, 1989), a simple example was given to illustrate the relation of the stochastic regime compared to the semiclassical and the quantum. We see that (at least for linear gravitational perturbations) the stochastic equations contain the same information as in quantum gravity, with the quantum average replaced by the noise average. (See also Anastopoulos (2001); Calzetta *et al.* (2003)). The difference is that for stochastic gravity the average of the energy momentum tensor is taken with respect only to the matter field, but not the graviton field. The important improvement over semiclassical gravity is that it now carries information on the correlation (and the related phase information) of the energy momentum tensor of the fields and its induced metric fluctuations which is absent in semiclassical gravity. (The relation between *fluctuations* and *correlations* is a variant form of the fluctuation–dissipation relation). The correlation in quantum field and geometry fully present in quantum gravity yet completely absent in semiclassical gravity, is partially captured in stochastic gravity. It is in this sense that a stochastic gravity gives an improved description and is closer to the quantum than the semiclassical.

Noise or fluctuations holds the key to probing the quantum nature of spacetime in this vein. The background geometry is affected by the correlations of the quantum fields through the noise term in the Einstein–Langevin equation, manifesting as induced metric fluctuations. The Einstein–Langevin equation in the form written down in Calzetta and Hu (1994); Campos and Verdaguer (1996); Hu and Matacz (1995); Hu and Sinha (1995); Lombardo and Mazzitelli (1997) contains only the lowest order term, i.e., the two-point function of the energy momentum tensor (which contains the 4th order correlation of the quantum field, or gravitons, when they are considered as matter source).[9] Noise in a broader sense embodies the contributions of the higher correlation functions in the quantum field. One could deduce generalized Einstein–Langevin equations containing more complex forms of noise, which fall under the same stochastic gravity programatic scheme. Progress is made on how to characterize the higher order correlation functions of an interacting quantum field systematically from the Schwinger–Dyson equations in terms of 'correlation noises' (Calzetta and Hu, 1995, 2000), similar to the classical BBGKY hierarchy.

One can generalize this scheme to the gravity–matter system, viewed as a system of strongly interacting fields, towards a description of the microstructure of spacetime. Starting with stochastic gravity we can get a handle on the correlations of the underlying field of spacetime by examining (observationally if possible,

[9] Although the Feynman–Vernon scheme can only accomodate Gaussian noise of the matter fields and takes a simple form for linear coupling to the background spacetime, the notion of noise can be made more general and precise. For an example of a more complex noise associated with more involved backreactions arising from strong or nonlocal couplings, see Johnson and Hu (2002).

e.g., effects of induced spacetime fluctuations) the hierarchy of equations, of which the Einstein–Langevin equation given in Calzetta and Hu (1994); Hu and Matacz (1995); Hu and Sinha (1995); Campos and Verdaguer (1996); Lombardo and Mazzitelli (1997) is at the lowest order, i.e., the relation of the mean field to the two-point function, and the two-point function to the four (variance in the energy momentum tensor), and so on. One can in principle move up in this hierarchy to probe the dynamics of the higher correlations of spacetime substructure. This is the basis for a correlation dynamics/stochastic semiclassical approach to quantum gravity (Hu, 1989, 1999).

2.2. Quantum Coherence in the Gravity Sector Obtained From Correlations of Induced Metric Fluctuations

Noise carries information about the correlations of the quantum field. One can further link *correlation* in quantum fields to *coherence* in quantum gravity. This linkage is ensured in principle, by virtue of the fact that at the quantum gravity level a complete quantum description should be given by a coherent wave function of the combined matter and gravity sectors. This linkage is operationally viable because of the self-consistency requirement (full backreaction is included) in the Einstein (classical level), the semiclassical Einstein (semiclassical level), and the Einstein–Langevin equations (the stochastic level) which relate the matter and spacetime sectors at the respective levels. Semiclassical gravity does not contain any information about the quantum coherence in the gravity sector. Stochastic gravity improves on the semiclassical in that it preserves partial information related to the quantum coherence in the gravity sector, by including the correlations in the matter field which contains quantum coherence information.

Since the degree of coherence can be measured in terms of correlations, our strategy towards quantum gravity in the stochastic gravity program is to unravel the higher correlations of the matter field, go up the hierarchy starting with the variance of the stress-energy tensor, and through its linkage with gravity (the lowest rung provided by the Einstein equation), retrieve whatever quantum attributes (partial coherence) of gravity left over from supra-Planckian high energy behavior. Thus in this approach, focusing on the noise kernel and the stress-energy tensor two-point function is our first step beyond the mean field theory (semiclassical gravity) towards probing the full theory of quantum gravity.

We have only addressed the correlation aspect; there is also the quantum to classical aspect. One way to address this issue is by the decoherence of correlation histories scheme proposed in Calzetta and Hu (1993), another is by the large N approximation.[10]

[10] In Hu (1999, 1989), I also brought up the relevance of the large N expansion in gravity for comparison. There exists a relation between correlation order and the loop order (Calzetta and Hu, 1995). One

2.3. Spacetime as an Emergent Collective State of Strongly Correlated Systems

At this point, it is perhaps useful to revisit an earlier theme we presented in the beginning, i.e., Stochastic semiclassical gravity as mesoscopic physics.

Viewing the issues of correlations and quantum coherence in the light of mesoscopic physics we see that what appears on the right hand side of the Einstein–Langevin equation, the stress-energy two-point function, is analogous to conductance of electron transport which is given by the current–current two-point function. What this means is that we are really calculating the transport functions of the matter particles as depicted here by the quantum fields. Following Einstein's observation that spacetime dynamics is determined by (while also dictates) the matter (energy density), we expect that the transport function represented by the current correlation in the fluctuations of the matter energy density would also have a geometric counterpart and equal significance at a higher energy than the semiclassical gravity scale. This is consistent with general relativity as hydrodynamics: Conductivity, viscosity, and other transport functions are hydrodynamic quantities. Here we are after the transport functions associated with the dynamics of spacetime structures. The Einstein tensor correlation function calculated by Martin and Verdaguer (1999a,b, 2000) is one such example. Another example is in the work of Shiokawa on mesoscopic metric fluctuations (Shiokawa, 2000).

For many practical purposes we do not need to know the details of the fundamental constituents or their interactions to establish an adequate depiction of the low- or medium-energy physics, but can model them with semi-phenomenological concepts. When the interaction among the constituents gets stronger, or the probing scale gets shorter, effects associated with the higher correlation functions of the system begin to show up. Studies in strongly correlated systems are revealing in these regards. Thus, viewed in the light of mesoscopic physics, with stochastic gravity as a stepping stone, we can begin to probe into the higher correlations of quantum matter and with them the associated excitations of the collective modes in geometro-hydrodynamics, the kinetic theory of spacetime meso-dynamics, and eventually, quantum gravity—the theory of spacetime micro-dynamics.

In seeking a clue to the micro-theory of spacetime from macroscopic constructs, we have focused here on the kinetic/hydrodynamic theory and

can also relate it to the order in large N expansion (see, e.g., Aarts and Berges (2002); Aarts *et al.* (2002)). It has been shown that the leading order $1/N$ expansion for an N-component quantum field yields the equivalent of semiclassical gravity (Hartle and Horowitz, 1981). The leading order $1/N$ approximation yields mean field dynamics of the Vlasov-type which shows Landau damping which is intrinsically different from the Boltzmann dissipation. In contrast, the equation obtained from the nPI (with slaving) contains dissipation and fluctuations manifestly. It would be of interest to think about the relation between semiclassical and quantum in the light of the higher $1/N$ expansions (Tomboulis, 1977), which is quite different from the scenario associated with the correlation hierarchy. The next to leading order calculation has recently been performed by Roura and Verdaguer (in preparation).

noise/fluctuations aspects. Another equally important factor is topology. Topological features can have a better chance to survive the coarse-graining or effective /emergent processes to the macro world and can be a powerful key to unravel the microscopic mysteries. This aspect is left for future discussions.

3. WHAT CAN WE LEARN ABOUT QUANTUM GRAVITY FROM BEC

In Section 1, we have stated the main theme of considering spacetime as a condensate, and mentioned several puzzles and challenges such a view evokes. We shall elaborate on those points in this section. But before doing so, we want to augment our physical intuition with a description of an analogy between phenomena observed in BEC collapse experiments (Claussen, 2003; Claussen *et al.*, 2003; Donley *et al.*, 2001), and quantum field processes in the early universe. This observation was made in a recent work (Calzetta and Hu, 2003).[11] The main features are described below.

3.1. Vacuum Cosmological Processes Found in Controlled BEC Collapse

We show that in the collapse of a Bose–Einstein condensate certain processes involved and mechanisms at work share a common origin with corresponding quantum field processes in the early universe such as particle creation, structure formation, and spinodal instability. Phenomena associated with the controlled BEC collapse observed in the experiment of Claussen (2003); Claussen *et al.* (2003); Donley *et al.* (2001) (they call it 'Bosenova,' see also Chin *et al.* (2003)) such as the appearance of bursts and jets can be explained as a consequence of the squeezing and amplification of quantum fluctuations above the condensate by the dynamics of the condensate.

The collapsing BEC is the time-reverse scenario of an expanding universe and the condensate plays a similar role as the vacuum in quantum field theory in curved spacetime. One can understand the production of atoms in the form of jets and bursts as the result of parametric amplification of vacuum fluctuations by the condensate dynamics. This is the same mechanism as cosmological particle creation from the vacuum, which is believed to be copious near the Planck time. Some basic ideas common to cosmological theories like "modes freeze when they grow outside of the horizon" can be used to explain the special behavior of jets and bursts ejected from the collapsing BEC. Finally the waiting time before a BEC starts to collapse obey a scaling rule which can be derived from simple principles of spinodal instability in critical phenomena.

Using the physical insight gained in depicting these cosmological processes, our analysis of the changing amplitude and particle contents of quantum excitations in these BEC dynamics provides excellent quantitative fits with the experimental

[11] We wish to mention other black hole (Barcelo *et al.*, 2001, 2003; Garay, 2000) and cosmological (Fedichev and Fischer, 2004) analog studies of BEC.

data on the scaling behavior of the collapse time and the amount of particles emitted in the jets. Because of the coherence properties of BEC and the high degree of control and measurement precision in atomic and optical systems, we see great potential in the design of tabletop experiments for testing out general ideas and specific (quantum field) processes in the early universe, thus opening up the possibility for implementing 'laboratory cosmology.'

3.2. What is a Condensate?

We have mentioned BEC as an example of a condensate. The spectrum is much broader. We now give a more systematic description of it. We will see the differences between photons and gravitons versus bosonic atoms in BEC; particles versus quasiparticles and collective excitations.

a. *Condensate as a "macroscopically populated" coherent state* Under this category are (i) "classical" electromagnetic wave, which can be thought of as a photon condensate; (ii) "classical" elastic wave as a phonon condensate; (iii) "classical" gravitational wave, a graviton condensate. Note that they are all coherent.
b. *Condensate as a non-trivial equilibrium phase at $T = 0$* This is the case for (i) BEC, as far as the bosonic atoms are concerned and (ii) BCS state, as far as the fermionic atoms are concerned, but NOT for gravitons. In fact, the situation for gravitons is similar to (i) photons, (ii) phonons, (iii) the quasiparticles in a BEC.

We can perhaps better understand the meaning of a condensate through the following questions:

α. "Can spacetime be considered a condensate from some microscopic (more fundamental) substructure, so that the metric and its perturbations correspond to collective variables and collective excitations?" (This is the picture behind Hu (1996)). This is similar to vibrational modes (phonons) in a lattice of atoms, vibrational and rotational modes of a nucleus (nuclear collective model), BEC quasiparticles or He^4 superfluid dynamics.

β. One can also think of the condensate as a nontrivial quantum state in terms of the microscopic constituents, such as in the string theory picture (see discussions below). This is similar to the ground states in (ii) BEC (iii) BCS (both involving non-trivial Bogoliubov transformations), but different from (i) phonon vacuum in a lattice (just normal modes). We need a microscopic theory to distinguish these two cases, or more information about the structures arising from graviton–graviton nonlinear interactions.

Finally we can ask,

1. "Is there any way to rule out the possibility that the graviton vacuum (for different background geometries) is a condensate in the sense of either

case (α) or (β) above? We cannot think of a way to do so yet. One should think harder to either substantiate or falsify this view.

2. Are there any hints suggesting that this is a possibility?—Maybe. Examples are:

   (i) Trans-Planckian modes in black holes horizons (Jacobson, 1991, 1994, 1999).
   (ii) Black hole atom (Ashtekar *et al.*, 1998; Bekenstein, 1974, 1994; Bekenstein and Mukhanov, 1995; Strominger and Vafa, 1996); Black hole quasi-normal modes (Barvinsky *et al.*, 2001a,b; Birmingham *et al.*, 2003; Corichi, 2003; Dreyer, 2003; Hod, 1998, 2003; Kunstatter, 2003; Louko and Makela, 1996; Oppenheim, 2004); and Black hole event horizon fluctuations (Barrabes *et al.*, 1999, 2000; ; Casher *et al.*, 1996; Ford, 2000; Parentani, 2001; Sorkin, 1997; Massar and Parentani, 2000; Ford and Svaiter, 1997; Wu and Ford, 1999; Hu *et al.*, 1999; Sinha *et al.*, 2003; Makela and Peltola, 2004).
   (iii) Cosmological constant problem.

   This viewpoint may provide a more natural explanation of the dark energy mystery: Why is the cosmological constant so small (compared to natural particle physics energy scale) today, and so close to the matter energy density?

Using these finer distinctions it is worthy to explore the implications and contradictions from this viewpoint of a spacetime condensate.

4. SPACETIME CONDENSATE VIEWPOINT: IMPLICATIONS ON BASIC PRINCIPLES

4.1. Comparison with the Proposals of Volovik

As mentioned in Section 1, the body of work by Volovik can probably be perceived as closest to our view here. For this reason it is perhaps useful to delineate the similarities and differences. Put broadly, we would say that the general philosophy and perspective are similar. (So is with Jacobson (1995)), but differences exist in the working principles or the choice of models. While we admire the boldness in Voloviks proposals, we would exercise caution in making certain sweeping claims. Nevertheless, the points of agreements are more basic and concordent:

1. Low energy properties of different vacua are robust: Magnets, superfluids, crystals, liquid crystals, superconductors. They do not depend much on the details of microstructure, i.e., atoms.
2. Microphysics only provides the constants of macrophysics: Speed of sound, superfluid density, modulus elasticity, magnetic susceptibility. In our view, these are all derived properties of an emergent structure. They are not fundamental in the sense that there are microscopic structures beneath.

3. Principal role played by symmetry and topology.
4. Different universality classes dictate different behaviors. One could in principle deduce the properties of vacuum energy, e.g., it is zero and non-gravitating (Weinberg, 1989).

4.2. Implications on Quantum Mechanics and General Relativity

The attitudes towards these two pillars of modern physics, quantum mechanics (QM) and general relativity (GR), are as varied as there are original thinkers. As a useful contrast, I mention two views very different from this one presented here: (1) The first group, represented by Penrose, is willing to give up quantum mechanics but holds on to GR; (2) Our view here regarding spacetime as an emergent entity in the low energy limit leads us to give up on GR beyond the Planck scale when the deeper level of microstructure of spacetime and matter reveals itself; and (3) the third group, spearheaded by Hooft (1999), views quantum mechanics not as a fundamental theory but as a set of bookkeeping rules.

(1) Those in the first group regarding general relativity as the deeper theory—more foundational and elemental—are ready to give up on quantum mechanics. In particular, Penrose consigns gravity the role of facilitating the decoherence of macroscopic quantum phenomena which shapes the classical world.

(2) In this view, GR is only an effective theory valid in the macroscopic limit. Lorentz invariance and gauge principles are emergent symmetries. Quantum mechanics governs the microstructures (atoms, strings) and, as expressed in this talk, may even govern the macrostructures, as collective phenomena. (quasi-particles, condensates).

(3) According to ‘t Hooft quantum mechanics should be viewed as dissipative classical dynamics. One apparent difficulty of this view is in the interpretation of dissipative processes (and the arrow of time issue) in the context of time-reversal invariant laws in relation to the basic tenets of statistical mechanics. One very interesting thought (to this author at least) is that quantum mechanics is a set of rules which provides an efficient bookkeeping scheme in our perception of the classical world.[12] This is a probe into the nature of quantum mechanics. If true, his viewpoint would demote the role of quantum mechanics from a fundamental theory of nature to a scheme, a clever scheme nonetheless, of bookkeeping. By no means does it imply quantum mechanics is ‘wrong’—because it has proven to work in the physical world we happen to live in.

[12] This position on the classical–quantum dichotomy is almost the reverse of the decoherence viewpoint. The adherents of quantum mechanics would consider the classical world as an emergent entity from the decoherence (be it environment-induced or consistent-history) of a microscopic world governed by quantum dynamics.

4.3. What is the Atom of Spacetime? Implications in Relation to String Theory and Loop Quantum Gravity

Let us now turn to the tough question:

4.3.1 "What is the Atom of Spacetime?"

How do we see or find them? In BEC the answer is obvious. BEC is made from atoms so it is not difficult or surprising to find atoms originating from, and interacting with, the BEC. Indeed, in the BEC experiments, when the vacuum (condensate) is squeezed by a controlled collapse, atoms appear in bursts or jets (see, e.g., Donley *et al.* (2001); Claussen (2003); Claussen *et al.* (2003)). But we should be mindful that not just atoms are produced: At a different magnetic field range, molecules are produced, as evidenced from Ramsey fringes of molecule-condensate resonances. At higher energies one can produce other energetic particles. Going beyond the confines of atomic physics, at nuclear energies one can think of quark gluon plasma and their condensates (Arsene *et al.*, 2005). At SUSY scales, one can think of Higgs condensates. String condensates, e.g., of ghosts (Arkani-Hamed *et al.*, 2004), if they exist, will also count as forms of matter structure, albeit at a much deeper level. Thus, there could be as many condensates as there are different levels of matter or particle structure they are made of.

At today's low energy the information of the detailed composition is grossly coarse-grained. Only the stress-energy tensor of matter is needed to determine the large-scale curvature of spacetime. Thus, one cannot attribute a unique type of condensate which makes up the spacetime macrostructure as we see it today. Condensates at all levels of matter structure can contribute, probably with a weighing factor depending on their spectral distribution which varies with energy.

A geometric description of spacetime is possible only in the low-energy long-wavelength limit. Beyond the hydrodynamic regime there may exist as many different mesoscopic regimes for spacetime structures as there are the corresponding condensates. None of the low-energy or ultra-low temperature condensates could, by themselves, reveal the atomic structure of spacetime. But maybe in the squeezing of the vacuum (as during rapid expansions of the early universe in analogy to the Bosenova experiment) or 'tearing up' the spacetime manifold (as in crossing shock waves or in black hole collisions) a deeper layer of structure may reveal. This is one of the motivations behind exploring possible kinetic theory regimes between the hydrodynamics (general relativity) of spacetime structure and the molecular dynamics of quantum gravity.[13]

[13] Castro and Granik (2003) claims that he knows what the 'Atoms or Quanta' of Spacetime are: "bubbles" (or p-loops) of hypervolumes.

4.3.2 Implications for String Theory: Can Spacetime be Derived from Strings

How does the basic premises of string theory fit into this picture? It does, in the sense that general relativity has been shown to be the low energy limit of string theories (Green *et al.*, 1990). Whether spacetime is the hydrodynamic limit of string theory has yet to be shown, but it is believed to be plausible (Herzog, 2002, 2003). This is not so trivial an issue as it may seem, because so far most discussions of string theory still assume a background spacetime where the strings propagate and interact. (String cosmology certainly makes such an assumption, when the line element of a FRW or de Sitter universe is written down). The real challenge is for the interacting strings to produce a spacetime, or at least to see spacetime emerge in some parameter range of their interactions. The advent of D-branes (Polchinsky, 1998) and duality relations greatly simplify and organize the structure of string theory, with five interconnected types, all manifestations of the one M-theory. Discovery of the AdS/CFT correspondence (Maldacena, 1998) changed the perspective and emphasis significantly. Now one can say that what happens at the gauge theory (CFT) sector has an exact correspondence in the spacetime (AdS) sector. In fact it is very interesting because one can find out the strong coupling regime of gravity from the weak coupling regime of gauge theories. But can one say that one sees the emergence of spacetime? Perhaps. Perhaps not, because the two different regimes are for two different entities (spacetime and gauge theory). The correspondence provides interesting and important connections of "known" physics, such as QCD and GR (e.g., deconfinement transition in QCD linked to Hawking–Page transition of black hole nucleation from thermal AdS space). New physics operative at trans-Planckian scales is still elusive. The belief is that if we know how to find solutions to more types of string theory or if we can formulate string field theory, new physics will appear. Or, perhaps there is no need or no room for trans-Planckian physics because of the IR/UV duality. I think it is fair to say that the structural relation of spacetime and strings remains an open question and a weighty issue.

4.3.3 Implications for Loop Quantum Gravity: Is Spin Connection a Fundamental or Collective Variable?

The discovery of the Ashtekar variables (Ashtekar, 1987) was viewed as an important step for solving the Einstein constraint equation in quantum general relativity. Indeed the focus is on quantizing the spin connection. Another important step in this program which lends its current name is in recognizing the significance of Wilson loops (Gambini and Pullin, 1996; Rovelli and Smolin, 1990) in the loop (Faraday) formulation of gauge theories. For recent developments, see Ashtekar and Lewandowski (2004); Nicolai *et al.* (2005); Rovelli (2004); Thiemann (2001).

Considering the quantization of gauge theories in relation to our view of GR as hydrodynamics, two questions naturally arise, one is for the loop gravity program: (1) Is the gauge connection a fundamental or collective variable? This has important implications in the true value of such a program of quantum gravity. The other question is for this geometro-hydrodynamics program. Since gauge theories share a similar structure as general relativity, if one regards the connection form in GR as a collective variable, how should one view it in gauge theories, such as the electromagnetic potential? (2) Would not one then regard all gauge bosons as collective variables and gauge symmetries as emergent properties particular to these variables? This is a daring challenge this program raises. For adherents of this program the logical answer to the second question should be YES. Then one would be faced with the difficult task of finding composite or emergent properties for what we would usually regard as ostensibly elementary particles, like photons. In this light, the recent proposal of string nets and quantum order by Wen is of unusual fundamental significance (Wen, 2003). According to his theory, the collective excitations in string-net condensed phase can behave just like light and electrons in our vacuum. This suggests that light and electrons as well as other elementary particles may originate from string-net condensation in our vacuum. This is a logical requisite of the idea that all gauge bosons (expressed in terms of connection forms) are collective entities. If string-net condensates are found, the discovery will lend strong support to the spacetime condensate idea, which will have far-reaching consequences in theoretical physics across the board.

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